Math Virtual Learning

Algebra IIB

The Data Unit - What is Normal?

May 8, 2020

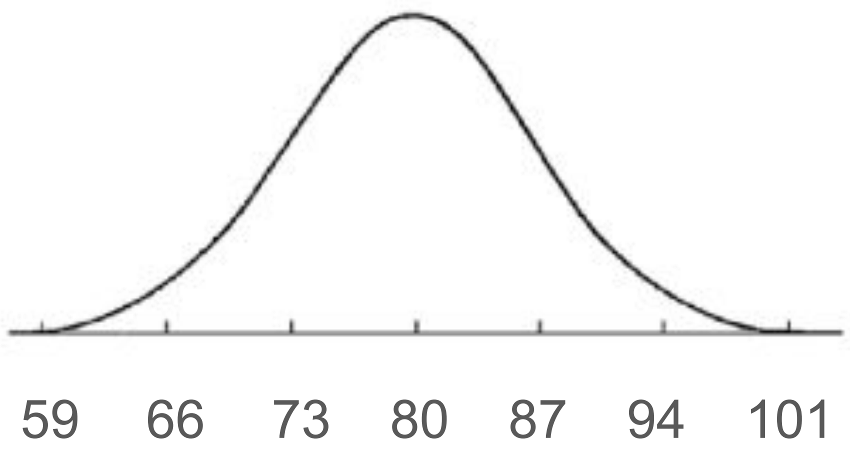
Algebra IIB
Lesson: May 8, 2020

Objective/Learning Target:

Students will be able to calculate the z-score of a set of data

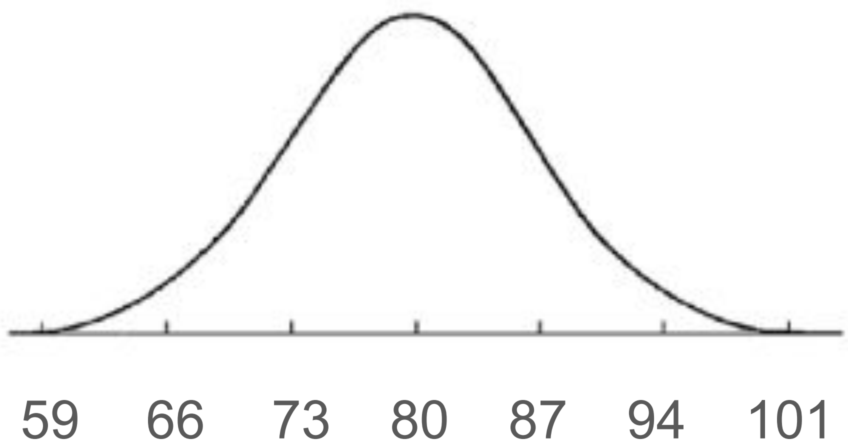
Let's Get Started!

Carlos and Mary were working on drawing some conclusions about the success of their students on a recent test. They mapped out this curve based on finding that the average test score was an 80% and the standard deviation was 7.



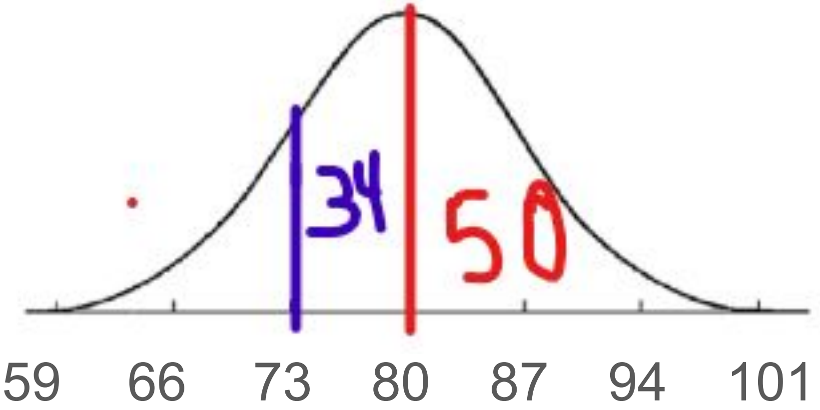
Let’s Get Started!

Carlos made the claim that 68% of the students scored at least a 73% on the test. Mary claims that it is actually higher; it is really 84% Who is correct?



Let’s Get Started! ANSWER

Carlos made the claim that 68% of his students scored at least a 73% on the test. Mary claims that it is actually higher; it is really 84% Who is correct?



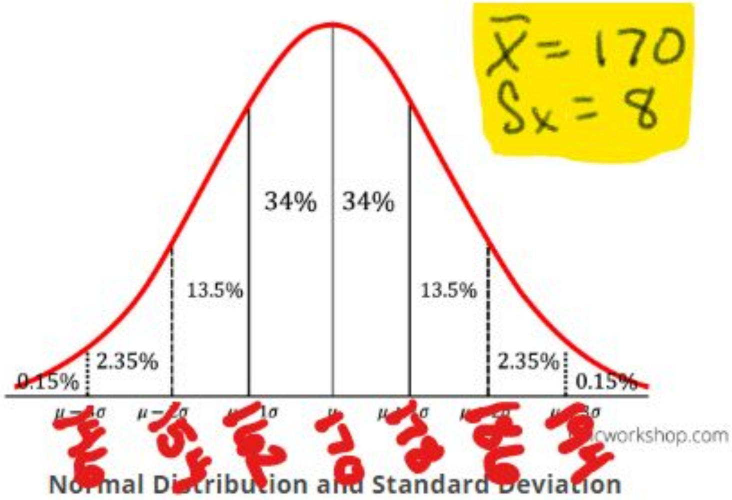
Mary is correct. 50% of the students scored 80 or above. Since 73 is one standard deviation away from the mean, that means that another 34% scored between 73 and 80.

This is a total of 84%

Z-Scores

We have been practicing with percentages when the number we are evaluating falls perfectly on a standard deviation. But what happens when it doesn't?

In the example on the right, you know what % is between 154 and 178 because 154 and 178 fall perfectly on a standard deviation. You can use the Empirical Rule to add the standard percentages to get an answer of 13.5+34+34 = 81.5%

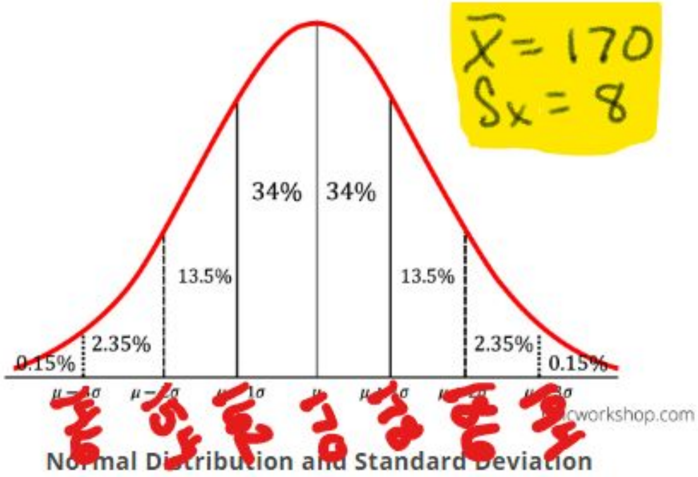


Z-Scores

But what happens if you need the percentage that falls between 154 and 172? Or the percentage that is below 180?

154 is on a perfect standard deviation, but 172 is not and neither is 180.

This is where we introduce the Z-Score formula to help you get an exact placement on the graph so that you can then use a chart tomorrow to find the exact percentages you need.



Z-Score Formula

Today we will focus only on the formula and how to calculate it.

In order to use the Z-Score Formula you will need 3 pieces of information:

1. The number you are looking for (x)
2. The Mean
3. The Standard Deviation

The Z-Score is going to tell you EXACTLY (in decimal form) how many standard deviations you are from the mean.

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$

From TopTipBio.com

Z-Scores

$$Z = \frac{\overbrace{x}^{\text{Score}} - \overbrace{\mu}^{\text{Mean}}}{\underbrace{\sigma}_{\text{SD}}}$$

***** Be sure that when you subtract on top, ALWAYS do your # minus the Mean!!** *(if you subtract it backwards you will get a wrong answer)*

In our example on slide 5 (Mean: 170 & SD: 8) we need to calculate the Z-Score for:

1. 154: $\frac{154-170}{8} = \frac{-16}{8} = -2$

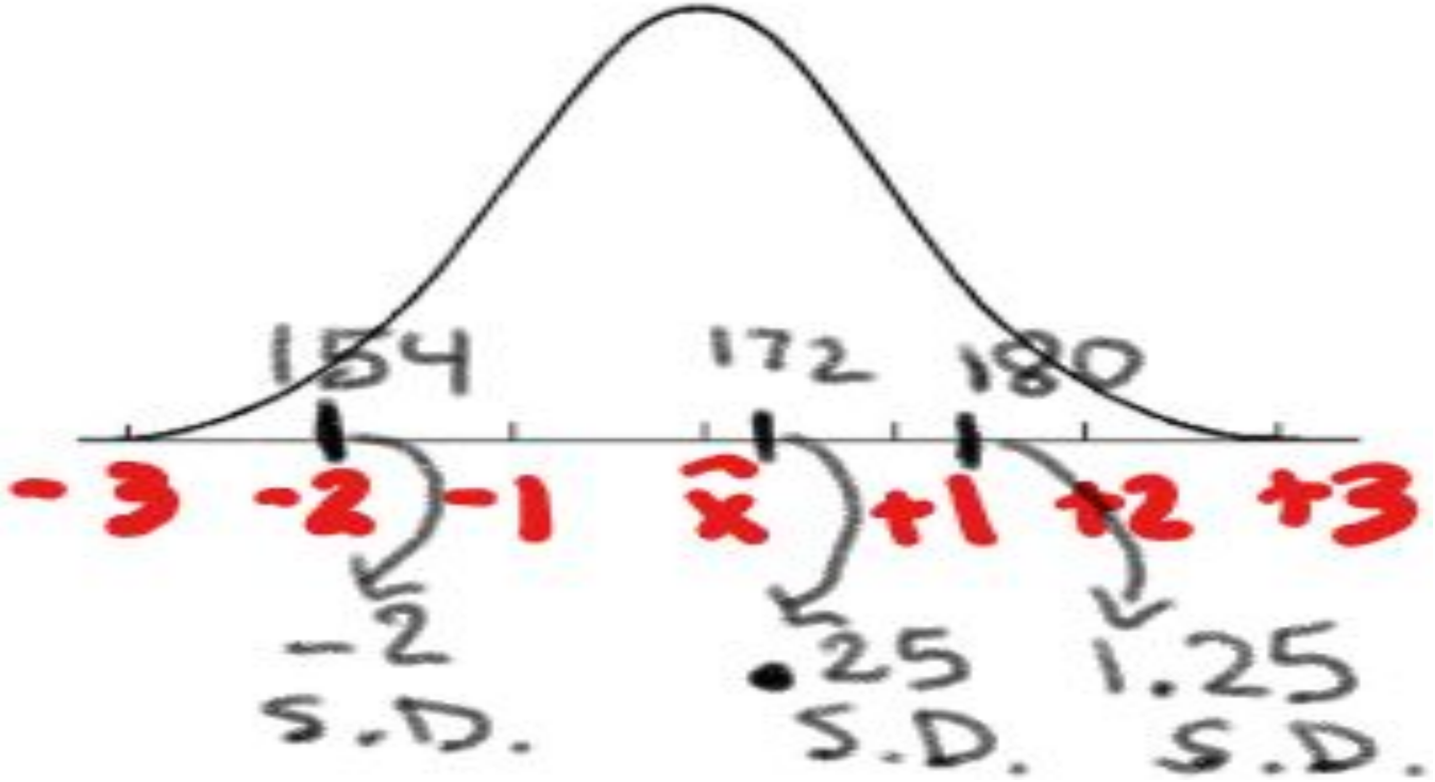
This answer should make sense because 154 is sitting at -2 (2 to the left) standard deviations from the mean on our bell curve graph.

2. 172: $\frac{172-170}{8} = \frac{2}{8} = .25$

This answer means that 172 falls .25 standard deviations to the right of 170 because it is positive .25 (¼ of the way between 170 & 178). This should make sense because 172 is closer to 170.

3. 180: $\frac{180-170}{8} = \frac{10}{8} = 1.25$

This answer means that 180 falls 1.25 standard deviations to the right of 170 because it is positive 1.25. If it had been negative it would fall to the left of the mean.



The Z-Score is going to tell you (in decimal form) where to put the tick mark on your graph

Video

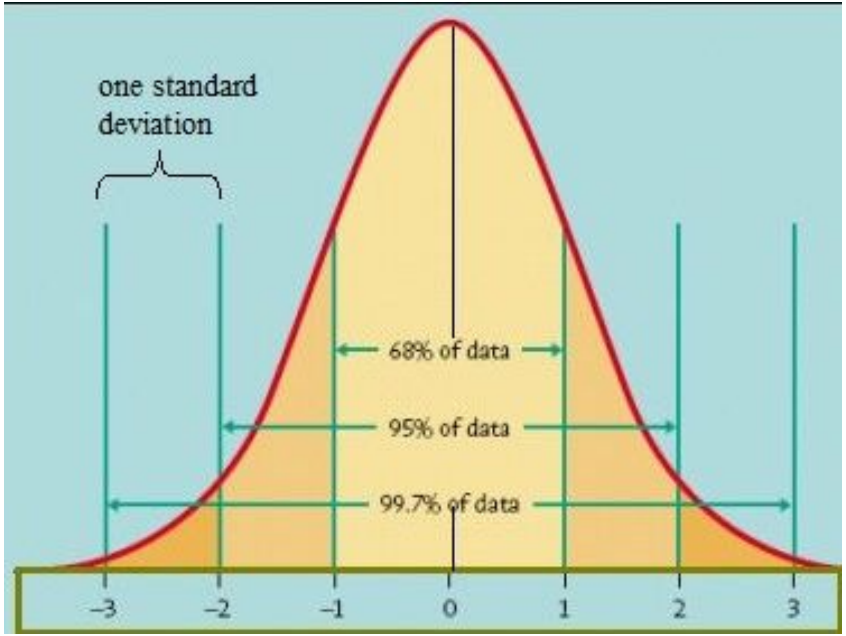
Watch this video to see more examples:

Z-Score Demo 1
(only watch section 3:10 to 4:30 of this video)

Z-Score Demo 2

Is the Data Point Normal?

- If the z-score falls between -2 standard deviations and +2 standard deviations from the mean, this data point is considered “Usual” or “Normal”.
- If the z-score falls between -3 and -2 or +2 and +3, this data point is considered “Unusual”.
- If the z-score is less than -3 or greater than +3, the data point is considered “Very Unusual”.



Working Backwards:

Sometimes you will be given the Z-Score and be asked to find the X value that corresponds to it.

It is basically an Algebra Problem and you are solving for X

This short video will walk you through it:

Z-Score Working Backwards (finding X)

Z-Scores and X-Values

*** Keep in mind that you will be asked to find both Z-Scores AND X values so be sure that you can work the formula forwards AND backwards.

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$

Practice Example #1

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. Suppose they want to know how many students scored a C (70%) or higher. Find the z-score of 70% and describe what this number means.

Practice Example #1 ANSWER

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. Suppose they want to know how many students scored a C (70%) or higher. Find the z-score of 70% and describe what this number means.

$$Z = \frac{70 - 80}{7} = -1.43$$

This means that 70 is 1.43 standard deviations to the left of the mean (because it's negative)

Because it is greater than -2, 70% would be considered a "normal" score.

Practice Example #2

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. What test score would have a z-score of 2.5?

Practice Example #2 ANSWER

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. What test score would have a z-score of 2.5?

$$2.5 = \frac{x - 80}{7}$$

-----> $17.5 = x - 80$ (multiply both sides by 7)

-----> $x = 97.5$ (add 80 to both sides)

On your Own

1. Nicole's score on the Algebra 2 midterm was 80 points. The class average was 75 and the standard deviation was 5 points
    a. What was her z-score?
    b. Would her score be considered usual, unusual or very unusual?
2. Cars currently sold in the US have an average of 135 horsepower with a standard deviation of 40 horsepower.
    a. What is the z-score for a car with 195 horsepower?
    b. Would the 195 horsepower be considered usual, unusual or very unusual?
3. The average score on the Algebra 2 final was 75 points with a standard deviation of 6 points. If Gregors z-score is -2.4,
    a. how many points did he score?
    b. Would her score be considered usual, unusual or very unusual?
4. People with z-scores greater than 2.5 on an IQ test are considered as geniuses. If IQ tests have a mean of 100 and standard deviation of 16 points, what is the cutoff score for a genius to prove himself as one?

On your Own

5. A town's January high temperatures for the month averaged 36 degrees with a standard deviation of 10 degrees, while in July the mean high is 75 degrees with a standard deviation of 8 degrees.

a. In which month is it more unusual to have a day with a high temperature of 55 degrees?
b. Why?

6. Scores on the ACT college entrance exam follow a Normal distribution with a mean 18 and a standard deviation 6. Wayne's standardized score(z-score) on the ACT was -0.7.

a. What was Wayne's actual ACT score?
b. Is this an unusual score?

7. A patient recently diagnosed with Alzheimer's disease takes a cognitive abilities test and scores a 45. The mean on this test is 52 and the standard deviation is 5. Is this score unusually low?

On your Own

8. Comparing batting averages. Three landmarks of baseball achievement are Ty Cobb's batting average of .420 in 1911, Ted William's .406 in 1941 and George Brett's .390 in 1980. These batting averages cannot be compared directly because the distribution of major league batting averages has changed over the years. The distributions follow a Normal curve, except for outliers such as Cobb, Williams and Brett. While the men batting average has been held roughly constant by rule changes and the balance between hitting and pitching, the standard deviation has dropped over time. Here are the facts:

Decade	Mean	Standard Deviation
1910s	0.266	0.0371
1940s	0.267	0.0326
1970s	0.261	0.0317

Compute the standardized batting averages (z-scores) for Cobb, Williams, and Brett to compare how far each stood above his peers.

On your Own

9. Three students take equivalent stress tests. Which is the highest relative score (meaning which has the largest z score value)?

a. A score of 144 on a test with a mean of 128 and a standard deviation of 34.

b. A score of 90 on a test with a mean of 86 and a standard deviation of 18.

c. A score of 18 on a test with a mean of 15 and a standard deviation of 5.

10. SAT vs. ACT. Eleanor scores 680 on the SAT mathematics test. The distribution of SAT scores is symmetric and single peaked, with a mean of 500 and standard deviation of 100. Gerald takes the American College Testing (ACT) Mathematics test and scores 27. ACT scores also follow a symmetric, single-peaked distribution- but with mean 18 and standard deviation 6.

a. Find the standardized scores (z-scores) for both students.
b. Assuming that both tests measure the same kind of ability, who has the higher score?

Answers to On Your Own Practice

1a. $z = (80-75)/5 = 1$
1b. Usual - only1 standard deviation away from the mean
2a. $z=1.5$
2b. Higher but since it is less than 2 still usual
3a. $-2.4=(x-75)/6$; $x=60.6$
3b. Unusually low because it is more than 2 standard deviations away from the mean
4. $2.5 = (x-100)/16$; $x = 140$
5a. $z_{Jan} = (55-36)/10 = 1.9$; $z_{June} = (55 - 75)/6 = -2.5$; it is more unusual for it to be 55 degrees in June.
5b. The z-score of -2.5 is farther away from 0 than the z-score of 1.9.
6a. $-0.7 = (x-18)/6$; $x = 13.8$
6b. No because -0.7 is close to 0.
7 $z = -1.4$; that is not unusually low because it is within 2 standard deviations of the mean.
8a. Cobb: $z = (0.42-0.266)/0.0371 = 4.15$
Williams: $z = (0.406-.0267)/0.0326 = 4.26$
Brett: $z = (0.390-0.261)/0.0317 = 4.06$
8b. All were very unusually good, but Williams was the best, followed by Cobb and Brett.
9. $z_1 = (144-128)/34 = 0.47$
$z_2 = (90-86)/18 = 0.22$
$z_3 = (18-15)/5 = 0.6$ This one was the highest relative score, although it is in the “usual” range.
10 $z_{SAT} = (680-500)/100 = 1.8$ Eleanor scored better although both scores are in the “usual” range.
$z_{ACT} = (27-18)/6 = 1.5$